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(Research Article)

Estimation of effective thermal conductivity for porous material

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Abstract

In this study, an algebraic expression for the effective thermal conductivity (ETC) of two-phase porous materials is derived using a two-layer model, which is analogous to the electrical resistance in an electrical circuit. This analytical approach enables the evaluation of the ETC through the proposed algebraic expression for two-dimensional porous media. The geometries under consideration include arrays of non-touching in-line and staggered square and circular cylinders, as well as touching hexagonal arrangements, rectangular grilles, criss-cross grilles, hexagons, and touching square cylinders. Comparison of the results obtained from the proposed algebraic expression with existing numerical solutions demonstrates excellent agreement.

Keywords: Effective Thermal Conductivity; Porous Material; Two Phase Materials; Different Geometrical Shapes

1. Introduction

The problem of determining the effective thermal conductivity of a porous medium has been great interest to chemical engineers and geophysicists for more than hundred years. Thermal conductivity of a porous medium can be calculated based on a two-layer model analogous to electric resistance in an electric circuit. The value of the effective thermal conductivity is maximum if the two layers are in parallel in the direction of the temperature gradient. On the other hand, if the two layers are in series in the direction of the temperature gradient, the effective thermal conductivity of the medium is minimum.

Thermal characteristics of porous depends upon various factors such as thermal conductivity of constituent phase, porosity shape factor, size of particles etc. according for all these factors to predict the effective thermal conductivity of a two-phase mixture is a complex affair. Engineering systems often include heterogeneous material such as composite parts, integrated electronics packages, and other solid bodies with inclusion of secondary materials.

Scientists and engineers have been interested in thermal conductivity of porous system since the end of the last century. By now vast experimental material has been accumulated and a lot of formulae have been proposed for the prediction of the effective thermal conductivity of porous system is due to the fact that they have found rather a wide use in a number of industrial branches.

A two-layer modal analogous to electric resistance in an electric circuit is used the essence of the method is to assume one dimensional heat conductional in a unit cell. The cell is then divided into parallel layers in the direction of the microscopic temperature gradient. The stagnant thermal conductivity of the composite layer is then obtained based on a series model. Result based on the algebraic expression obtained from these two dimensional porous media are found in excellent agreement with the solution obtained by Behrens as well as L.H.Han. In addition; In case of inline touching square cylinder, we obtained a non –dimensional parameter for maximum conductivity.

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2. Effective Thermal Conductivity

The Effective Thermal Conductivity of a composite material depends on the following parameters:

- The Thermal Conductivities of the constituent phase (solid and fluid phase)
- The volume concentration of the two phases (1-ϕ) and ϕ

- Where ϕ is the fractional porosity

$$\text{Porosity}(\phi) = \frac{volume\ of\ voids}{total\ volume}$$

- The distribution of the two phases in the material and it will also depend upon the particle of pore Size. At high temperature when radiative heat transfer in gases fluid is not negligible.
- Shape factor: it is observed that the theoretical expression Based on rigid geometry do not represent the true state of affairs in a real system.

2.1. Theory

Thermal conductivity of a porous medium can be calculated based on the two-layer model an analogous to electric resistance in an electric circuit.

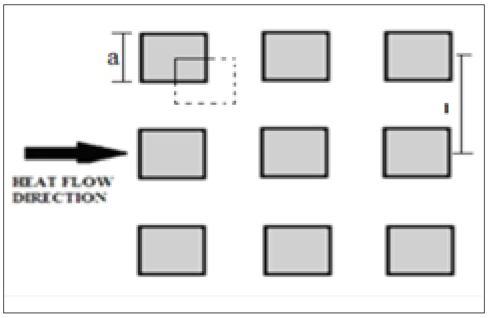
An application of the electric resistance analogy will lead to algebraic expression for the effective thermal conductivity of a number of two-dimensional periodic porous media. The essence of the method is to assume one dimensional heat conduction in a unit cell. The cell is then divvied in to parallel layers (which may consist of fluid, solid or composite layers) in the direction of the microscopic temperature gradient. The effective thermal conductivity of the composite layer is then obtained on an series model. The analysis is based on the following assumptions for a unit cell of the regular periodic porous materials.

2.1.1. These assumptions are

- The heat flux is unidirectional in the X direction.
- No transfer of heat occurs by way of convection or radiation.
- The contact resistance between the solid and fluid phase is negligible.
- Thermally, the mixture is isotropic.

3. The effective thermal conductivity of different shapes with different arrangements

3.1. Square cylinder



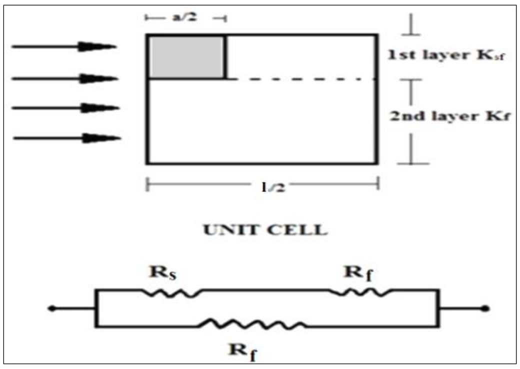


Figure 1 Two dimensional line arrays of square cylinders

Consider two dimensional in line arrays of square cylinders, as shown in fig 1 because of the symmetry of thermal paths ,we choose a unit cell. It can be considered to be consisting of two layers.

I layer: A rectangular composite layers consisting of both solid and fluid phases.

II layer: A rectangular fluid phase only.

The electrical resistance circuit diagram of unit cell is as shown in fig 1

Porosity of square cylinder is $\varphi = 1 - Y_a^2$

The resistance (R) of the circuit is related to the thermal conductivity (K) by the following relation.

$$R = \frac{L}{KA}$$

Where, L is the length in the direction of temperature gradient

A is the cross sectional area

The effective thermal conductivity of the two parallel layers is

$$\frac{k_e\ A}{L} = \frac{k_{sf}\ \ A_{sf}}{L_{sf}} + \frac{k_f\ \ A_f}{L_f}$$

$$k_e = k_{sf}\frac{a}{l} + k_f(l - \frac{a}{l}) \quad \ldots\ldots\ldots\ldots \quad (1)$$

The value of k_{sf} can be obtained on a series layer model. It is given by ,

$$\frac{L_{sf}}{k_{sf}\ \ A_{sf}} = \frac{L_s}{k_s\ \ A_s} + \frac{L_f}{k_f A_f}$$

$$\frac{k_{sf}}{k_f} = \frac{1}{Y_a\lambda\ + (l - Y_a)} \quad \ldots\ldots\ldots.(2)$$

Where Y_a= a/l

Substituting the equation (2) into equation (1) yields

$$\frac{k_e}{k_f} = 1 - Y_a + \frac{Y_a}{1 + Y_a(\lambda - 1)}$$

Nomenclature:

k_e= effective thermal conductivity of the composite medium
k_f= thermal conductivity of the fluid phase
k_s= thermal conductivity of the solid phase
k_{sf}= equivalent thermal conductivity of a composite layer
$l/2$= length of unit cell
a= length of one side of the solid surface cylinder.
λ =fluid/solid thermal conductivity ratio.
ϕ=porosity

The same value of $\frac{k_e}{k_f}$ is obtained in its staggered form. Similarly we derived the ETC of different shapes in-line array and staggered arrangements. The unit cell of each arrangement is shown bellow.

3.2. Circular cylinder

Consider two dimensional in line arrays of regular periodic circular cylinders as shown in fig 2. Its unit cell consist two layers.

Porosity of circular cylinder is $\varphi = 1 - \frac{\pi}{4} Y_a$

The effective thermal conductivity of two parallel layers are

$$k_e = k_{sf}\frac{a}{l} + k_f\frac{(l-a)}{l}$$

Where,

a= radius of circular cylinder , Y_a= a/l

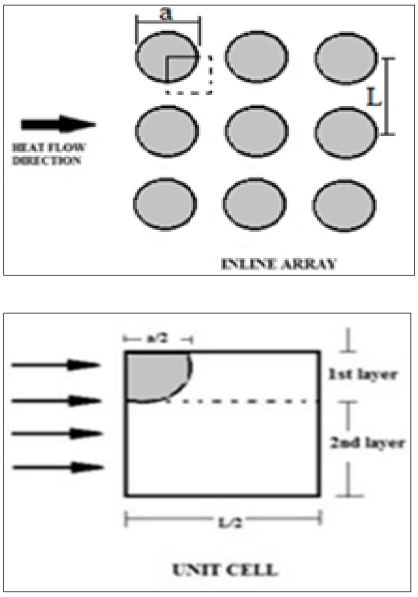


Figure 2 Two dimensional line arrays of Circular cylinder

The value of k_{sf} is obtain on a series layer model, so

$$\frac{k_e}{k_f} = 1 - Y_a + \frac{\pi}{2(\lambda-1)} - \frac{2}{(\lambda-1)\sqrt{1-Y_a^2(\lambda-1)^2}}\frac{\tan^{-1}\sqrt{1-Y_a^2(\lambda-1)^2}}{1+Y_a(\lambda-1)}$$

3.3. Rectangular grill

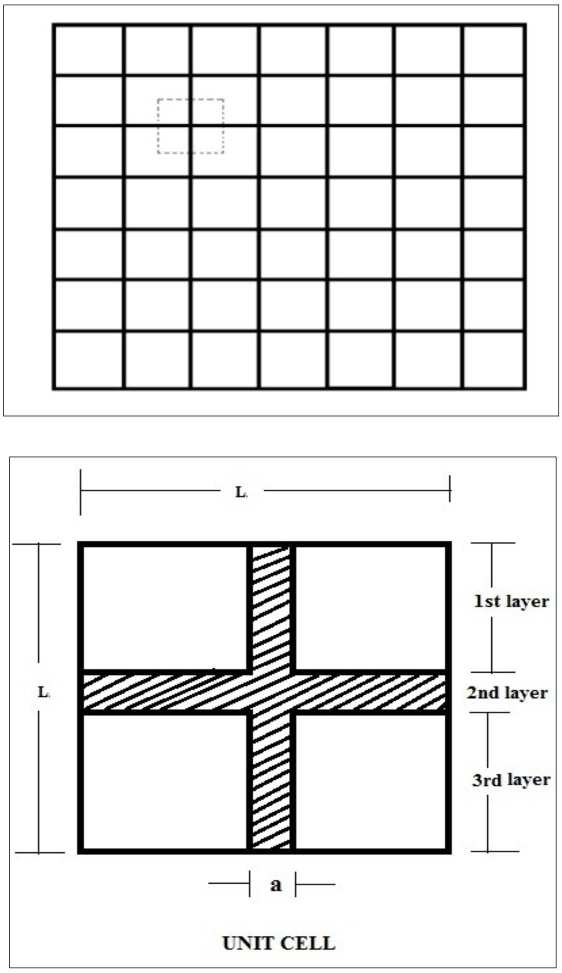


Figure 3 Two dimensional line arrays of Rectangular grill

For rectangular grill porosity

$$\varphi = 1 - Y_a^2 - 2Y_a$$

Its unit cell consist of three layer , the effective thermal conductivity of three parallel layers are:

$$k_e = \frac{\frac{k_{sf_1}(\mathrm{l}-\mathrm{a})}{2}}{\mathrm{l}} + k_s \frac{\mathrm{a}}{\mathrm{l}} + \frac{k_{sf_3}(\mathrm{l}-\mathrm{a})/2}{\mathrm{l}}$$

k_{sf_1} and k_{sf_3} are the thermal conductivity of first and third layers respectively.

Its value is obtained by series layer model

so

$$\frac{k_e}{k_f} = \frac{1 - Y_a}{(1 - Y_a) + \lambda Y_\mathrm{a}} + \frac{Y_a}{\lambda}$$

3.4. Criss cross grill

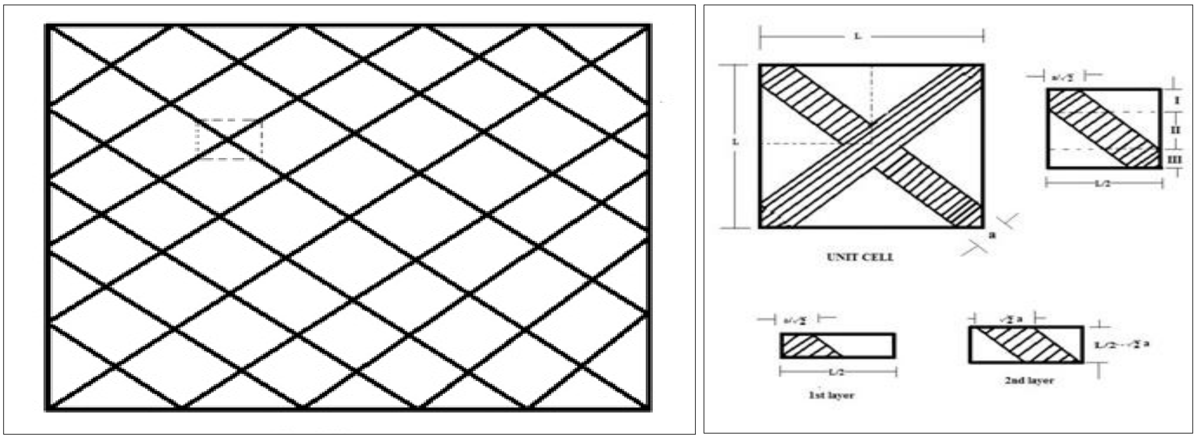


Figure 4 Two dimensional line arrays of Criss cross grill

The effective thermal conductivity of porous cross grill is

$$\frac{k_e \mathrm{l}}{2} = 2k_{sf_1}\frac{\mathrm{a}}{\sqrt{2}} + k_{sf2}\frac{\mathrm{l}}{2} - \frac{2\mathrm{a}}{\sqrt{2}}$$

Where ,

a= thickness of the grill

k_{sf_1} and k_{sf_2} are the thermal conductivity of first and third layers respectively.

Its value is obtained by series layer model

So the value of $\frac{k_e}{k_f}$ is

$$\frac{k_e}{k_f} = \frac{2}{(\lambda-1)}\ln\left[\frac{1+2\sqrt{2}Y_a(\lambda-1)}{1+\sqrt{2}Y_a(\lambda-1)}\right] + \frac{1+2\sqrt{2}Y_a}{1+2\sqrt{2}Y_a(\lambda-1)}$$

3.5. Hexagon

Consider the periodic porous hexagon as shown in fig 5 Because of symmetry of thermal paths. The unit cell is divided into four equal parts by two perpendicular line at its canter. The quadrants of unit cell consist of three layers.

Composite resistance of I layer.

$$R_1 = \frac{2l\cos\theta - a}{k_f\dfrac{2l\cos\theta - a}{2\cos\theta}} + \frac{\lambda\, a}{k_f\dfrac{2l\cos\theta - a}{2\cos\theta}}$$

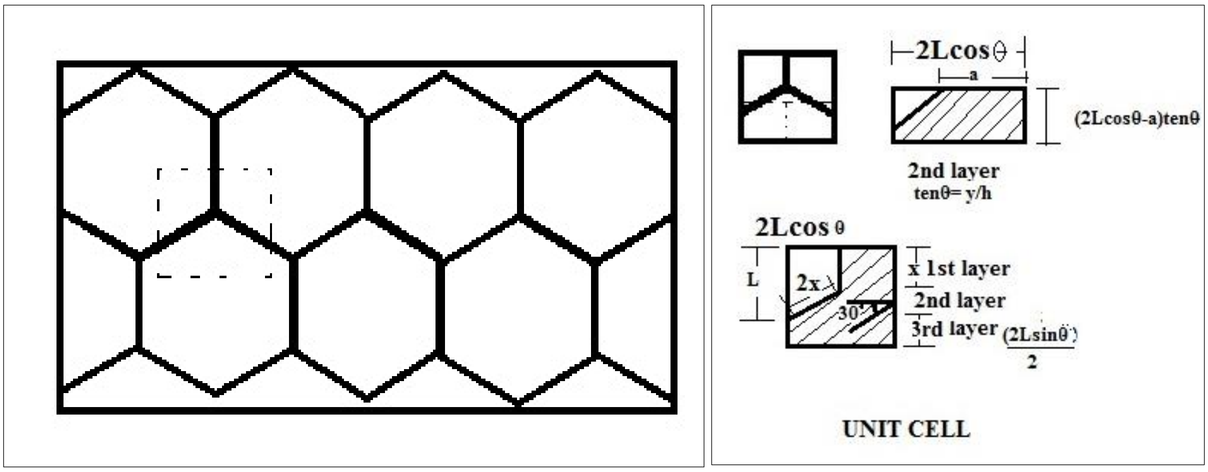


Figure 5 Two dimensional line arrays of Hexagon

Composite resistance of II layer

$R_2 = \left(k_f \frac{\tan\theta}{(1-\lambda)} \ln\left\{1 + \frac{(1-\lambda)}{\lambda}(1-Y_a)\right\}\right)^{-1}$ Composite resistance of III layer

$$R_3 = \frac{2l\cos\theta}{k_f\left(\frac{\mathrm{a}}{\cos\theta} - 0.5\ \mathrm{l}\right)}$$

So effective resistance of all three parallel layers is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Substituting the value of $R_1, R_2\ , R_3$

Where

$$Y_a = \frac{\mathrm{a}}{2\mathrm{l}\cos\theta}$$

So

$$\frac{1}{R} = \frac{k_f(2l\cos\theta - a)}{2l\cos\theta + a\,(\lambda - 1)2\cos\theta} + k_f\frac{\tan\theta}{(1-\lambda\,)}\ln\left\{1 + \frac{(1-\lambda\,)}{\lambda}(1-Y_a)\right\} + \frac{k_f\left(\frac{\mathrm{a}}{\cos\theta} - 0.5\ \mathrm{l}\right)}{2l\cos\theta\ \lambda}$$

4. Touching square cylinder

Consider The effective thermal conductivity of two dimensional in line arrays of square cylinders having squer cross section area of $(a \times a)$ with connecting plates having a width of c,, as shown in fig 6 because of the symmetry of thermal paths ,we choose a unit cell. It can be considered to be consisting of three layers.

: A rectangular solid layers and two rectangular composite layers consisting of both solid and fluid phases. The dimensions of three layers are $(\frac{c}{2} \times \frac{l}{2})$ for solid layer and $\frac{l/2}{(a-c)/2}$, $\frac{l/2}{(l-a)/2}$ for composite layer .

The area solid fraction of unit cell is

$$1 - \varphi = Y_a^2 + 2Y_aY_c(1 - \mathrm{Y_a}) \dots\dots\dots \quad (1)$$

Where,
Y_a=a/l

Y_c=c/a
L=length of unit cell
a=length of one side of the solid square cylinder
c=width of connecting plate
φ=porosity

The effective thermal conductivity of the three parallel layer is

$$k_e = Y_a Y_c k_s + Y_a(1-Y_c)k_{sf1} + (1-Y_a)k_{sf2} \quad \dots\dots.. \quad (2)$$

k_{sf_1} and k_{sf_2} are the thermal conductivity of composite layers.

Its value is obtained by series layer model ,for example , the value of k_{sf_1} is given by

$$\frac{l}{k_{sf1}} = \frac{a}{l\, k_s} + \frac{l-a}{l\, k_f}$$

This can be written as

$$\frac{k_{sf1}}{k_f} = \frac{1}{1+Y_a(\lambda-1)} \quad \dots\dots \quad (3)$$

Where,

λ =fluid/solid thermal conductivity ratio.

Similarly the value of k_{sf2} is

$$\frac{k_{sf2}}{k_f} = \frac{1}{1+Y_a Y_c(\lambda-1)} \quad \dots\dots\dots\dots \quad (4)$$

Substituting equations (3) and (4) into equation (2) yields

$$\frac{k_e}{k_f} = \frac{Y_a Y_c}{\lambda} + \frac{Y_a(1-Y_c)}{1+Y_a(\lambda-1)} + \frac{(1-Y_a)}{1+Y_a Y_c(\lambda-1)} \quad \dots\dots\dots (5)$$

Now for a given value of $\lambda = 0.5$the effective thermal conductivity $\frac{k_e}{k_f}$ is maximized with variable parameter Y_a *and* Y_c. then the equation (5) becomes

$$\frac{k_e}{k_f} = 2Y_a Y_c + \frac{Y_a(1-Y_c)}{1-0.5Y_a} + \frac{(1-Y_a)}{1+.5Y_a Y_c} \quad \dots\dots.. \quad (6)$$

Let $1-\varphi = N$

Therefore equation (1) can be written as

$$Y_a Y_c = \frac{N - Y_a^2}{2(1-Y_a)} \quad \dots\dots \quad (7)$$

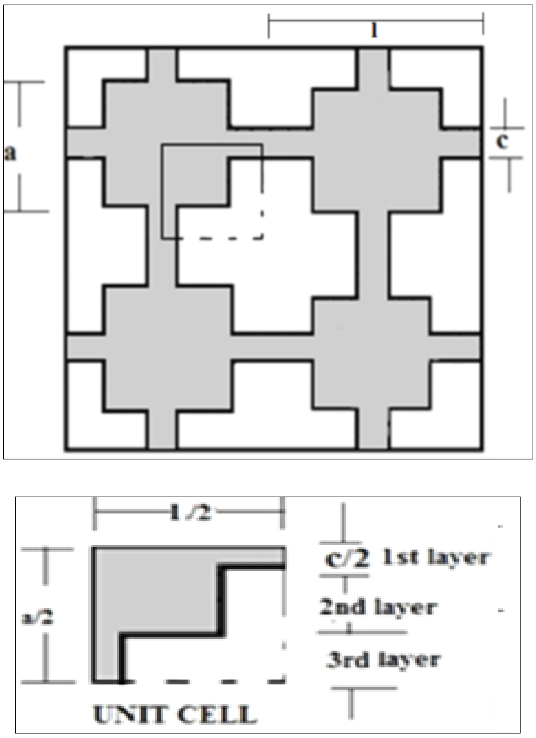


Figure 6 Two dimensional line arrays of Touching square cylinder

Substituting the value of $Y_a Y_c$ from equation (7) $\frac{k_e}{k_f} = \frac{\mathrm{U}}{\mathrm{V}}$ ……… (8)

Where

U = $4 + 2\mathrm{N} - 0.5\ \mathrm{N}^2 + Y_a(-10 - 5\mathrm{N} + 0.5\mathrm{N}^2) + {Y_a}^2(8 + 4\mathrm{N}) + {Y_a}^3(-1 - \mathrm{N}) + {Y_a}^4(-1.5) + 0.5{Y_a}^5$

And

$\mathrm{V} = 4 + \mathrm{N} + Y_a(1.5\mathrm{N}^2 - 10) + {Y_a}^2(9 - .5\mathrm{N}) + {Y_a}^3(3.5\mathrm{N}) + 0.5{Y_a}^4$

This id equivalent to ..

$$\frac{A + BY_a + \mathrm{C}{Y_a}^2 + \mathrm{D}{Y_a}^3 + \mathrm{E}{Y_a}^4 + \mathrm{F}{Y_a}^5}{G + HY_a + \mathrm{I}{Y_a}^2 + \mathrm{J}{Y_a}^3 + \mathrm{K}{Y_a}^4}$$

Where A,B,C,D…K are consists as given in equation (9). Differentiating $\frac{k_e}{k_f}$ with respect to Y_a

And equating to zero .

$$\frac{d}{dY_\mathrm{a}}\left(\frac{k_e}{k_f}\right) = 0$$

$= .25{Y_a}^8 - 3.5{Y_a}^7 + 14.75{Y_a}^6 + (0.5 - 55\mathrm{N}){Y_a}^5 + (-0.25\mathrm{N}^2 + 3.75\mathrm{N} + 89){Y_a}^4 + (1.5\mathrm{N}^2 - 16\mathrm{n} - 82){Y_a}^3 + (0.25\mathrm{N}^3 - 3.25\mathrm{N}^2 + 24\mathrm{N} + 40){Y_a}^2 + (-0.5\mathrm{N}^3 + 3\mathrm{N}^2 - 16\mathrm{N} - 8)Y_a + 0.25\mathrm{N}^3 - \mathrm{N}^2 + 4\mathrm{N} = 0$ (10)

This equation is solved by computer added programme.

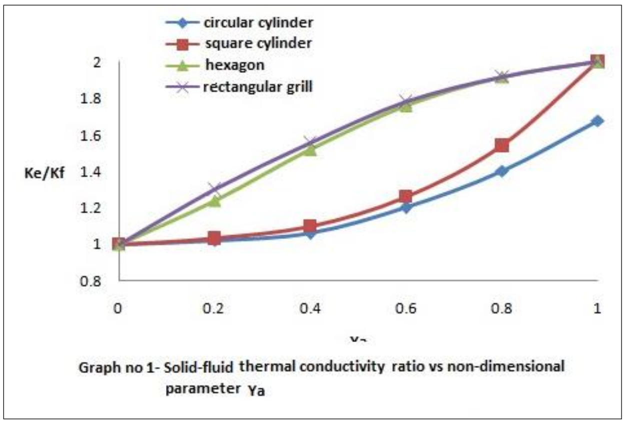


Figure 7 Solid Fluid Thermal Conductivity Ratio Vs Non Dimensional Parameter Ya

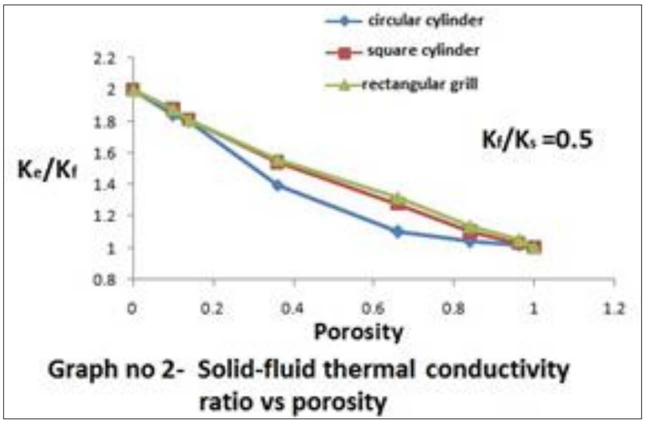


Figure 8 Solid Fluid Thermal Conductivity Ratio Vs porosity

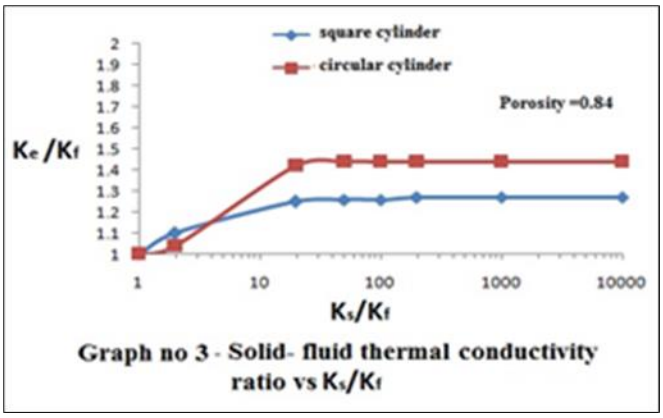


Figure 9 Solid Fluid Thermal Conductivity Ratio Vs K_S/K_f

5. Results and discussion

A theory of heat conduction in two phase porous media has been presented based on the analogy of electric resistances in an electric circuit. The effective thermal conductivities of two phase porous media of different shapes have been calculated employing our model. The results are presented in a Figure.

Figure no 7 shows the effect of particle geometry. The effect of the particle shape on the effective thermal conductivity is examined by using square cylinders, circular cylinders, rectangular grill and hexagon. A curve of effective thermal conductivity versus non-dimensional parameter y_a is shown for the given value of λ as 0.5. The Figure reflects that effective thermal conductivity of rectangular grill shape has higher value of thermal conductivity. However, square and inclined square shapes also have same optimum and minimum values of ETC at intermediate values of y_a.

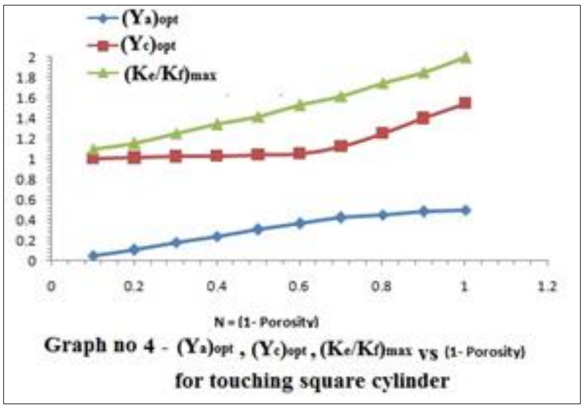


Figure 10 (ya) opt, (yc)opt and (ke/kf)max (1 - porosity) for touching square cylinder

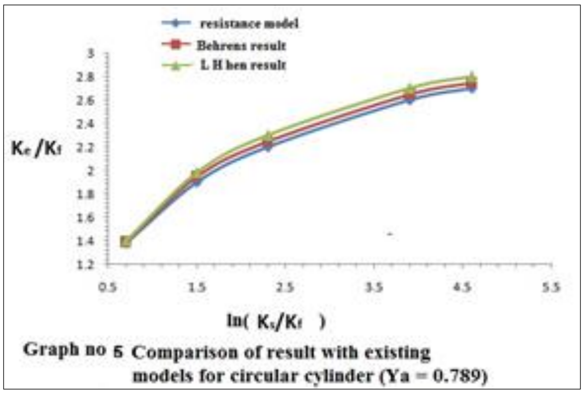


Figure 11 Comparison Of Result With Existing Models For Circular Cylinder (Ya Is Equal To 0.789)

- A similar plot of effective thermal conductivity (ETC) versus porosity shown in Figure no 8. The curves show that the effective thermal conductivity decreases as the porosity increases. This is due to the presence of increased void space.
- Figure no 9 shows the variation of effective thermal conductivity against solid- fluid thermal conductivity ratio. The high value of k_s-k_f ratio does not influence the ETC. It is important to note that the plot is a semi-log plot and porosity has been fixed at 0.84 in this case.
- Figure no 10 exhibits nature of maximize ETC for touching square cylinder. The corresponding values of (k_e/k_f) maximum and y_a are plotted against all value of N which is (1-ø). Similarly y_c is also plotted. The plot is thus between a scale of 0.0 to 2.0 and N for$(y_a)_{opt}$, $(y_c)_{opt}$ and $(k_e/k_f)_{max}$.
- Figure no 11 shows the comparison of model under consideration and Behrens result L.H. Han results for circular cylinder at y_a = 0.789. It is apparent that this model is in good agreement with their models.

6. Conclusion

In this work an attempt has been made to derive algebraic expression to obtain effective thermal conductivity of two-phase composite materials. The results obtained show good agreement with existing models. Also, effort has been made to find non-dimensional parameters y_a and y_c for maximum thermal conductivity considering the typical case of touching square cylinder.

Another important aspect is that in all the cases heat flow has been considered as a one dimensional only. The multi-dimensional analysis is complex and beyond the purview and scope of this work

This work has unlimited dimensions. To name a few, calculation of non dimensional parameters Y_a and Y_c for various shapes and orientations, multidimensional and determination of ETC of various shapes experimentally.

Compliance with ethical standards

Acknowledgments

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